AN APPLICATION OF ISO-GUM IN THE METHOD FOR ESTIMATING THE DIMENSIONAL ERRORS OF BENT PARTS

T. H. M. Nguyen[1], J. R. Duflou[2], J.-P. Kruth[2]
[1] Hanoi University of Technology, Vietnam, [2] Katholieke Universiteit Leuven, Belgium
Contact email: NguyenThiHongMinh@mail.hut.edu.vn

The current study addresses the accuracy of bend elements and global dimensions with tolerances based on an analytical method, where each deviation in part dimension is represented by type A and type B errors, as instructed by ISO-GUM. Meanwhile, a generic formula is presented to estimate the deviations in the global dimensions of interest in bent parts based on the deviations in the participating bend elements. The deviations in the bend elements formed in each operation are in turn estimated through an analytical simulation model. Therefore, the error propagation in consecutive bending operations using a specified process plan is simulated. Using such a top-down analytical approach, the errors of the global dimensions with tolerances were represented and estimated through the errors encountered for each operation. Finally, a procedure for estimating dimensional errors in bent parts is suggested and illustrated with an example to show the applicability of the method.

1. INTRODUCTION

Forming using sheet bending introduces straight bend lines into flat patterns cut out from metal sheets in order to form three-dimensional parts with various bend features for a wide range of part complexity and weight. Workpiece preparation for the process includes modelling the part, calculating and cutting out the corresponding flat pattern. Geometric approximations and material behaviour assumptions are applied in both modelling and calculation. Due to the discrepancies between the model and reality, there exists an error in the calculation of the unfolding (Streppel *et al.*, 1993). The subsequent cutting step may also provide various dimensional precision, depending on the selected cutting process. During forming, the workpiece is positioned against a backgauge of the machine before being bent linearly by a punch penetrating into the die cavity. The accuracy of the resulting dimensions depends on the repeatability of the gauging system, the gauging method, the estimation of the location of the gauging edge according to the process plan (de Vin *et. al*., 1996), the material handling method (Nguyen *et al*., 2005), the accuracy of the punch positioning (Singh *et al.*, 2004).

Considering the accuracy aspects for global dimensions of bent sheet metal parts, studies employing deterministic approaches (de Vin *et al.*, 1996) and Monte-Carlo simulations (Hagenah, 2003) have been conducted. The relationships between the causes and the effects of the inaccuracies have been investigated in order to estimate the achievable dimensional accuracy. A deterministic view of the tolerances (de Vin *et al.*, 1996) allows a fast estimation of the achievable tolerance zones, but the stochastic characteristics of the process cannot be taken into account. In contrast, Monte-Carlo simulation incorporates this aspect, with the computation time sacrificed.

2. ERROR ANALYSIS MODEL

According to studies reported in (de Vin *et al.*, 1996), (Hagenah, 2003), (Nguyen *et al.*, 2005), (Singh *et al.*, 2004) and (Streppel *et al.*, 1993), the dimensions achieved from bending operations suffer from various sources of errors. As instructed by (ISO, 1997), once a quantity Q is expressed by a function of n normally distributed variables in such form as in (1), then the deviation ΔQ of the quantity Q from the nominal value Q^0 can be expressed through a combination of the influencing errors, as expressed in (2)

$$Q = f_Q\left(X_0,..,X_k,..,X_n\right) \tag{1}$$

$$\Delta Q = \Delta(Q) \pm k_p \times u_c(f_Q) \tag{2}$$

$\Delta(Q)$ is the correctible systematic error of Q, representing errors that are known and can be compensated for, as classified by (ISO, 1997). According to (ISO, 1997), the value of ΔQ is estimated through the errors ΔX_k of the influencing factors k by Formula (3).

$$\Delta(Q) = \sum_{k=0}^{2n_I+1} \Delta(Q_k) = \sum_{k=0}^{2n_I+1} c_{Q,k} \times \Delta X_k \tag{3}$$

k_p is the coverage factor to obtain a confidence interval having level of confidence p assuming a normal distribution of Q. According to (ISO, 1997), k_p takes a value of 2 for $p = 95.45\%$.

$u_c(f_Q)$ is the combined uncertainty of quantity Q, covering all the uncertainties introduced by the unknown systematic errors and the random errors of the influencing factors. The value of $u_c(f_Q)$ is estimated through the standard uncertainty $u(X_k)$ of the influencing factors k by Formula (4).

$$u_C^2(f_Q) = \sum_{k=0}^{2n_I+1} u_k^2(f_Q) = \sum_{k=1}^{2n_I+1} c_{Q,k}^2 \times u^2(X_k) \tag{4}$$

The standard uncertainty, or deviation, $u(X_k)$ of random errors are generally estimated by statistical analysis. Meanwhile, the unknown systematic errors are generally estimated using ‘expert knowledge’ by assuming a range for the corresponding errors and the most likely statistical distribution, from which a standard deviation is estimated.

$c_{Q,k}$ is the sensitivity coefficient of element k in function f_Q, calculated by (5).

$$c_{Q,k} = \partial f_Q / \partial X_k \tag{5}$$

Therefore, when the errors of the participating elements and the sensitivity of function f_Q for each element k are known, the error of quantity Q, which is expressed through a relationship function as expressed in Formula (1), can be estimated by Formula (2). Hence, in order to estimate the errors expected for the dimensions of bent parts using this model, there are two tasks to be carried out: The first task is to find the expression of the bent dimensions through the influencing factors, and the second task is to find the errors of the influencing factors. These aspects are dealt with in the following sections.

3. MODELLING OF ERROR PROPAGATION

In (Nguyen *et al.*, 2005), the influence of the various uncertainties on the angular and linear bend elements resulting from a bending operation have been quantified. Besides, according to (de Vin *et al.*, 1996) and (Shpitalni *et al.*, 1999), dimensional errors of bent parts propagate in the bending process from one bending operation to the others. Meanwhile, the error expected for a global dimension is influenced by the dimensions participating in the dimension chain. In order to investigate these phenomena, this study divides the dimensions of bent parts into two groups: 1) Simple dimensions are the dimensions specified for a single bend element, such as the angle between two adjacent flanges, or the length of a bend leg between two adjacent bend lines. 2) Complex or global dimensions are the dimensions specified for a *partition* that consists of a group of adjacent bend elements. The following subsections provide the modelling of error propagation and stack-up in bent parts through two models: The first model in Section 3.1 provides the geometric functions of the global dimensions of a partition through its participating elements. Applying the error analysis model presented in Section 2 for such functions, the errors expected for the global dimensions can be calculated through the errors of the participating elements. Meanwhile, in Section 3.2, the second model investigates the propagation of the errors through the consecutive bending operations. By using the second model, the errors expected for each bend elements can be calculated to provide the data for the first model.

3.1. Error stack up on a global dimension

Among the various types of complex dimensions that can be specified for a bent part, the models for the two most common types are explained: the linear distance from a point to a flange and the angle between two non-adjacent flanges. The modelling uses the following assumptions: (1) All bend lines involved in a partition with tolerances are parallel to one another and the dimensions with tolerances are specified in cross sections perpendicular to the bend lines. (2) Bend radius and sheet thickness are not taken into account, as extended foil models (Duflou *et al.*, 2005) are used to represent bent parts. (3) The base points of dimensional tolerances are defined on the edges of bend flanges.

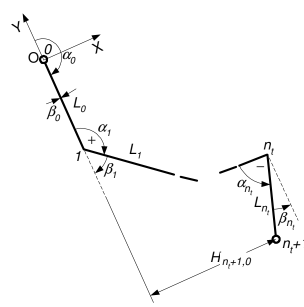


Figure 1. Linear dimension between a point and a flange.

Linear distance between a point and a flange. Take a partition of n_t bend lines, where a linear distance $H_{n_t+1,0}$ is specified between a point on the edge (n_t+1) and the first flange L_0. In the coordinate system fixed to flange L_0 (Figure 1), the projections L_i^x and L_i^y of each linear bend element L_i are calculated as in (6):

$$L_i^x = L_i \sin\left(\sum_{j=0}^{i} \alpha_j - i\pi\right) \text{ and } L_i^y = L_i \cos\left(\sum_{j=0}^{i} \alpha_j - i\pi\right) \tag{6}$$

i is the index of the edges of the flanges participating in the partition. Therefore, i equals 1 to n_t corresponds to the bend lines in the partition, while $i=0$ or n_t+1 correspond to the outer edges of the partition;

α_i is the angle formed at bend line i;

L_i is the bend length of flange i, which lies between edge i and $i+1$;

α_0 is the angle between +Oy of the current coordinate system and the flange L_0

By selecting the coordinate system so that $Oy \equiv L_0$ ($\alpha_0 = \pi$), distance $H_{n_t+1,0}$ is calculated through the bend elements participating in the dimension according to (7).

$$H_{nt+1,0} = \sum_{i=0}^{n_t} L_i \sin\left(\sum_{j=0}^{i} \alpha_j - i\pi \right) \quad (7)$$

Complex angular dimensions. A complex angular dimension $\varphi_{n_t,0}$ is defined by the angle formed between two non-adjacent flanges L_{n_t} and L_0. The dimension represents the change in the direction vectors $\vec{L_0}$ and $\vec{L_{n_t}}$ between the two flanges, as shown in Figure 2. From the figure, it can be seen that:

$$\varphi_{n_t,0} = \beta_{n_t} - \beta_0 \quad (8)$$

where $\beta_0 = (\pi - \alpha_0)$ and

$$\beta_i = \beta_{i-1} + (\pi - \alpha_i) \quad (9)$$

Thus $$\varphi_{n_t,0} = n_t \times \pi - \sum_{i=1}^{n_t} \alpha_i \quad (10)$$

Formula (10) expresses the angular dimension as a function of angular bend elements locating between the flanges defining the dimension. Therefore, once the angular bend elements are known, the complex angular dimension can be calculated. Note that the angular dimension is not influenced by any linear bend element.

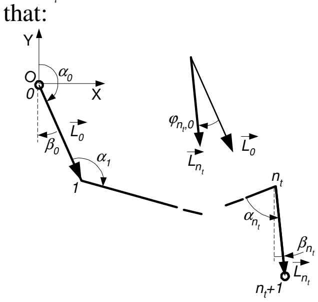
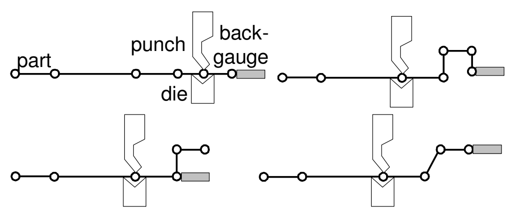


Figure 2. Complex angular dimension.

2.1 Error propagation in the bending process

The errors are propagated in the process through the sequence in which the bend lines of the part are made, the intermediate part shapes and the selection of the gauging options. For each operation, placing a workpiece against backgauge(s) positions the bend line to be bent at the designated position. The edge of the part that contacts with the backgauge is referred to as *the gauging edge*. Considering the relative position of the gauging edge to the position of the current bending line recognises two possible gauging options: 1) Direct gauging: no performed bend line between the gauging edge and the current bending line, as shown in Figure 3.a. 2) Indirect gauging: the gauging edge is connected to the current bending line through at least one performed bend line, as illustrated in Figure 3.b. When compared to the direct gauging situation, indirect gauging induces extra gauging errors to the bending operation. As illustrated in Figure 4, for each bend step s where bend i is performed by gauging to a line g, the error starts to propagate from the gauging line g towards the bend line i and finally stops at edge k, with k being the nearest bend line to i which is already bent at the operator side (OS). Therefore, the errors introduced into the operation influence only the actual unfolded length, locating between two edges k and j, where j the nearest bend line to i which is already bent at the machine side (MS). The errors do not have an impact on the *backgauge partition* $G_{j,g}$ standing between edges j and g, or the partition between the free edge at the OS and edge k.



(a) direct gauging (b) indirect gauging

Figure 3. Examples of gauging option

Meanwhile, the cumulative error on the backgauge partition contributes to the total gauging error introduced to the bend lengths produced by the bending operation.

From Figure 4, the length of the bend leg before bending at the MS is calculated by Formula (11). Note that, since the gauging movement occurs in the X direction of the machine coordinate system (MCS), only the error component in X direction actually influences the gauging accuracy.

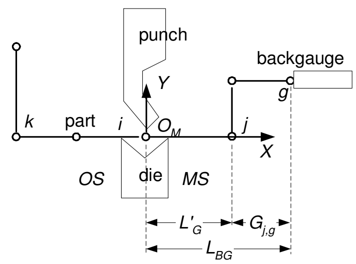


Figure 4. A complex model with indirect gauging

$$L'_G = L_{BG} - G^x_{j,g} \qquad (11)$$

where, L_{BG} is the distance between the current bend line and the actual backgauge and $G^x_{j,g}$ is the length of the projection in X direction of the gauging partition. Meanwhile, gauging against edge g introduced an intrinsic error ΔL^0_G to the nominal value L^o_{BG} of distance L_{BG}, as expressed in (12).

$$L_{BG} = L^o_{BG} + \Delta L^0_G \qquad (12)$$

Similarly, an intrinsic error ΔL^0_R is also introduced to the residual length at the operator side. Both ΔL^0_G and ΔL^0_R suffer from the unavoidable uncertainties participating in the process such as the material handling method, the machine factor, the bend allowance and the punch displacement, as reported in (Nguyen *et al.*, 2005). Therefore, for step s, the total gauging error due to the combined effect of the intrinsic error and the additional error due to gauging is calculated through:

$$\Delta L_{G|s} = \Delta L^0_G - \Delta G^x_{j,g} \text{ and } \Delta L_{R|s} = \Delta L_{U|s} + \Delta L^0_R + \Delta G^x_{j,g} \qquad (13)$$

$\Delta L_{G|s}$ and $\Delta L_{R|s}$ are the errors expected for linear bend elements at the MS and OS,

ΔL^0_G and ΔL^0_R are the intrinsic errors encountered at the MS and OS for direct gauging,

$\Delta L_{U|s}$ is the error in the unfolded length $L_{j,k}$ due to cutting process, as for the first bending operation, or due to previous bending operations.

$\Delta G^x_{j,g}$ is the additional error due to indirect gauging, which is the X component of the error expected for $G_{j,g}$. According to Section 3.1, the global dimensions of this partition can be estimated through its participating elements. Therefore, according to Section 2, the error $\Delta G^x_{j,g}$ can be calculated through those of the bend elements of the backgauge partition.

Thus, after a step s is executed, the resulting linear bend elements at the machine side and the operator side are estimated as expressed in Formula (14).

$$L_{G|s} = L^0_{G|s} + \Delta L_{G|s} \text{ and } L_{R|s} = L^0_{R|s} + \Delta L_{R|s} \qquad (14)$$

$L_{G|s}$ is the linear bend element formed between the current bend line i and edge j, the nominal value of which is $L^0_{G|s}$.

$L_{R|s}$ is the linear bend element formed between the current bend line i and edge k, the nominal value of which is $L^0_{R|s}$.

Note that for each production condition, the intrinsic errors as well as the errors of the initial unfolded length can be measured in advance. By repeated use of Formulas (13) and (14) from the unfolded state till the completely folded state of the part, the formation of all the linear bend elements is simulated. For any step with indirect gauging option, the additional gauging error is calculated through the errors attached to the bend elements participating in the gauging partition. Therefore, all the final bend elements can be calculated with their expected errors, providing a set of data to estimate the errors for any global dimension specified for the part.

4. PROCEDURE FOR ERROR ESTIMATION

The previous sections have provided the theoretical background to estimate the error for a dimension imposed with tolerance in a bent part. Complementary to the theory, the following paragraphs illustrate the procedure for such estimation through a real case.

4.1. Representation of the part using a foil model

A part with complex dimensions is given with the bend sequence as shown in Figure 5. In order to make use of the models presented in the previous sections, the part is converted into a foil model using the mid-planes of the original flanges. Two complex dimensions D_3 and φ_2 are selected to demonstrate the procedure.

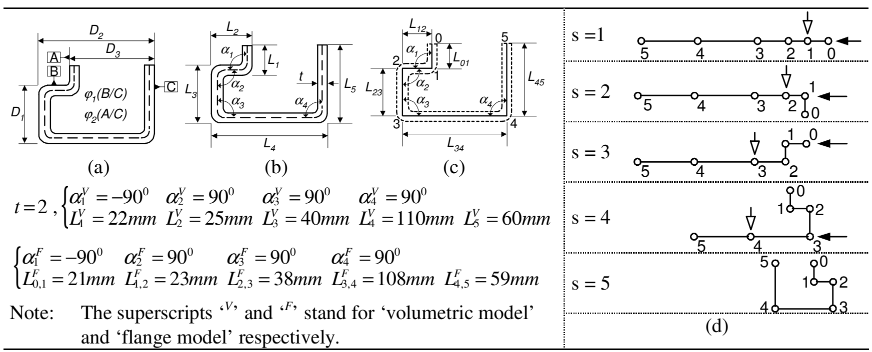


Figure 5. Original part description (a) with dimensions of interest, (b) part representations in volumetric model, (c) foil model and (d) the selected bend sequence and gauging edges for each bending step

4.2. Estimation of all bend elements and the corresponding errors

For estimating the errors expected for all the bend elements, the formation of the bend elements through all the bending steps is simulated according to the bend sequence. The simulation starts with the intrinsic errors and the error of the unfolded length measured for 6 series produced on 6 machines, coded M1 to M6, as shown in Table 1. Among all the bend steps, only step 3 has indirect gauging, where an additional gauging error is calculated through the errors estimated for the previously formed bend elements. As result, Table 2 shows the errors estimated for each bend element through two components: the standard deviation and the average value shift.

Table 1. The intrinsic errors measured for 6 series of parts produced on 6 machines.

Error type	$\Delta\alpha$ [deg]		$\Delta L_{U/0}$ [mm]		ΔL_G^0 [mm]		ΔL_R^0 [mm]	
Machine	stdev	avg	stdev	avg	stdev	avg	stdev	avg
M1	0.510	0.366	0.021	- 0.088	0.166	0.147	0.160	- 0.672
M2	0.095	0.422	0.032	- 0.080	0.042	- 0.006	0.102	0.766
M3	0.123	0.328	0.022	- 0.090	0.029	0.033	0.046	0.185
M4	0.161	0.586	0.023	- 0.058	0.029	- 0.029	0.035	0.078
M5	0.110	0.314	0.019	- 0.080	0.039	0.051	0.044	- 0.006
M6	0.096	0.102	0.027	- 0.083	0.022	0.046	0.030	0.201

Table 2. Simulation result - estimating the errors of the bend elements.

bend step	s = 1		s = 2		s = 3				s = 4		s = 5	
element	L_{01}		L_{12}		G_{20}		L_{23}		L_{34}		L_{45}	
machine	stdev	avg	stdev	avg	stdev	avg	stdev	avg	stdev	avg	stdev	avg
M1	0.093	0.147	0.093	0.147	0.112	0.281	0.145	-0.134	0.093	0.147	0.222	-2.494
M2	0.051	-0.006	0.051	-0.006	0.073	0.149	0.089	-0.155	0.051	-0.006	0.145	3.133
M3	0.025	0.033	0.025	0.033	0.061	0.153	0.066	-0.120	0.025	0.033	0.093	0.805
M4	0.057	-0.029	0.057	-0.029	0.078	0.186	0.097	-0.215	0.057	-0.029	0.148	0.438
M5	0.065	0.051	0.065	0.051	0.084	0.166	0.106	-0.115	0.065	0.051	0.160	0.060
M6	0.022	0.046	0.022	0.046	0.056	0.083	0.061	-0.037	0.022	0.046	0.093	0.802

4.3. Estimation of the error for the global dimensions

Investigating the foil model allowed expressing the complex dimensions of interest, namely D_3 and φ_2, through their participating bend elements as follows:

- D_3 is a complex linear dimension defined from point 0 to point 5 in the coordinate system of the reference flange $L_{4,5}$. Therefore, according to (7),

$$\begin{cases} D_3 = D_{0,5}^x = & L_{4,5}\times\sin(\alpha_0) + L_{3,4}\times\sin(\alpha_0+\alpha_4-\pi) + L_{2,3}\times\sin(\alpha_0+\alpha_4+\alpha_3-2\pi) + \\ & L_{1,2}\times\sin(\alpha_0+\alpha_4+\alpha_3+\alpha_2-3\pi) + L_{0,1}\times\sin(\alpha_0+\alpha_4+\alpha_3+\alpha_2-\alpha_1-4\pi) \end{cases} \quad (15)$$

- φ_2 is a complex angular dimension between flange $L_{0,1}$ and $L_{4,5}$, therefore according to Formula 10, $\varphi_2 = \varphi_{L_{0,1},L_{4,5}} = 4\times\pi - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4$ (16)

In order to apply the error analysis model discussed in Section 2, the sensitivity coefficients $c_{Q,k}$ are calculated for each pair of quantity Q, being either D_3 or φ_2, and the influencing factor, being any of the variables in the functions expressed by Formulas (15) or (16). Plugging the errors of the participating bend elements from the simulation result (Table 2) and the sensitivity coefficients calculated from Formulas (15) and (16) into Formulas (3) and (4) allowed estimating both components of the errors expected for the global dimensions of interest.

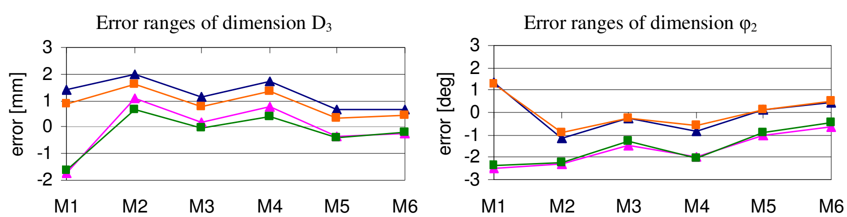


Figure 6. Graphical comparison of the $\pm 2\sigma$ error ranges for the dimensions of interest between the measured values (■: max, ■: min) and the values estimated (▲: max, ▲: min).

In order to validate the result, these dimensions are also measured on the real parts and the corresponding ranges of deviations are calculated for each series. Figure 6 illustrates the comparison between the estimated and the measured values of the errors expressed by the error ranges calculated for $\pm 2\sigma$. It can be seen from the graphs that the ranges of the estimated and measured errors have good agreement in most cases. Furthermore, it has been observed from the result, which has been used to plot these graphs, that the uncertainty component of the linear dimensions shown by the standard deviations has been predicted with an absolute error ranging between 0.01 to 0.06 mm. Occasionally, the estimation error can reach a very high value, e.g. 0.17 mm for machine M1, or a very low value, e.g. 0.01mm for machine M2. The average estimation error over all machines is 0.05 mm. The standard deviations of the angular dimensions have been predicted with an absolute error ranging mainly between 0.03 to 0.05 degrees. The estimation errors can seldom reach values as high as 0.08 degrees for machine M4, or as low as 0.03 degrees for machine M5. The average error over all machines is 0.05 degrees. On the other hand, the average offsets of the linear dimensions are predicted with an average error of 0.26 mm. Most of the prediction errors for the average offset range between 0.20 to 0.30 mm. This prediction error is occasionally low, i.e. with a value of 0.08 mm, for machine M6. For the angular dimensions, the average offsets are predicted with an average error of 0.08 degrees. For most of the cases, the prediction errors range between 0.02 to 0.07 degrees.

5. CONCLUSION

The study has established a method for estimating the dimensional errors for bent parts. Making use of the GUM method, it has been shown that such task can be tackled by two subtasks: the first step is establishing an analytical expression for the dimensions of interest through the participating bend elements, while the second subtask is to calculate the errors expected for these elements. The subtasks are correspondingly resolved by two mathematical models. Finally, the different pieces of the puzzle are fixed together through a procedure for estimating dimensional errors illustrated for an example. Validating the data achieved from the procedure against the measured data from real production has shown promising results for various series of parts produced by different machines. In brief, the study has shown the possibility to use simple analytical models in combination with experimental data to predict the dimensional errors in low time complexity yet with a useful level of accuracy. Such a prediction can be applied in automatic process planning sytems to judge the compatibility of the alternative process plans against the tolerance imposed for bent parts.

6. ACKNOWLEDGEMENT

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